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HIGHER APPROXIMATIONS TO THERMAL BOUNDARY-LAYER IN AN INVISCID FLOW

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Abstract—Second-order thermal boundary-layer problem is formulated for an inviscid plane or axisymmetric flow. The resulting energy equations which govern the effects of both longitudinal and transverse curvatures, external temperature gradient and vorticity are solved both exactly and approximately. Applications of the present theory are given for heat transfer from elliptical-rod bundles and that from a sphere.

NOMENCLATURE

$B(\psi)$, Bernoulli function defined by (11);
$H(\psi)$, temperature in outer region;
j, variable defined to be zero for plane flow and one for axisymmetric flow;
k_l, longitudinal curvature parameter defined by (23);
k_t, transverse curvature parameter defined by (24);
k_x, vorticity parameter defined by (25);
k_z, external temperature gradient parameter defined by (26);
L, characteristic length of the body;
Nu, local Nusselt number;
p, non-dimensional pressure referred to ρU_r^2;
Pe, Péclet number ($= U_r L/\alpha$);
q, heat flux at the wall;
$\mathbf{q}$, non-dimensional velocity;
r, non-dimensional radial coordinate;
r_0, non-dimensional body transverse radius of curvature;
t, non-dimensional temperature;
t_0, t_1, non-dimensional first- and the second-order temperature in outer region;
T_0, T_1, non-dimensional first- and the second-order temperature in inner region;
T_w, non-dimensional temperature at the surface of the body;
T_∞, non-dimensional free stream temperature;
T_r, characteristic temperature;
u, v, non-dimensional tangential and normal velocity components;
u_p, non-dimensional velocity at the surface of the body;
U_r, characteristic velocity;
U_∞, non-dimensional free stream velocity;
x, non-dimensional coordinate along the body;
y, non-dimensional coordinate normal to the body;
Y, stretched normal coordinate defined by (8).

Greek symbols

α, thermal diffusivity;
γ, principal thermal function defined by (32);
ε, small parameter defined by (35);
$\varepsilon_t, \varepsilon_x$, small parameter defined by (61);
ζ, variable defined by (20);
θ_0, θ_1, functions defined by (21);
κ, longitudinal surface curvature of the body;
ξ, variable defined by (20);
π_i, principal function defined by (32);
ρ, density;
ψ, non-dimensional stream function defined by (5).

1. INTRODUCTION

IN CALCULATING heat transfer to liquid metals or heat/mass transfer from a droplet, we can reasonably approximate the velocity in the energy equation by inviscid flow [1, 2]. This approximation usually makes analysis considerably simple, particularly when the flow is two-dimensional and irrotational, for which the well-known Boussinesq transformation may be successfully applied to the energy equation and the exact solutions of the transformed equation have already been obtained by Aichi [3] and also by Tomotika and Yosinobu [4]. However, exact solutions given by the former are of inconvenient forms for numerical calculation and that by the latter is applicable only to the case of constant wall temperature. Moreover, when the flow is three-dimensional, the Boussinesq transformation is of no use and exact solutions are still hopeless. Therefore, most of the investigators who calculated heat transfer in an inviscid flow introduced some further approximations, among which a thin thermal boundary-layer approximation is most frequently used. It is well known that the thermal boundary-layer theory represents the leading term in an asymptotic expansion of the energy equation for large Péclet numbers. For viscous flow, van Dyke [5] formulated systematically a set of both momentum and energy equations which govern the second-order boundary-layer effects, that is, the effects of displacement thickness, both longitudinal and transverse curvatures, external vorticity and stagnation enthalpy gradient, for the case of plane or axisymmetric flow and, after that, many investigators obtained solutions of these second-order equations. For inviscid flow, on the other hand, no attempt has been made, as far as the present writer knows, to formulate such a second-order problem.

The aim of the present paper is to formulate the first- and the second-order equations for steady incompressible inviscid flow past plane or axisymmetric solid bodies with nonisothermal wall temperature, and to give solutions of the resulting equations, both exactly and approximately. To obtain the approximate solutions, an approximate method proposed by Merk [6] for calculating non-similar viscous boundary layer is extended.

As examples of applications of the present theory, heat transfer from elliptical-rod bundles and from a sphere is calculated.

FORMULATION OF THE PROBLEM

Consider steady incompressible inviscid flow past plane or axisymmetric solid bodies as shown in Fig. 1, in which all lengths are referred to a characteristic

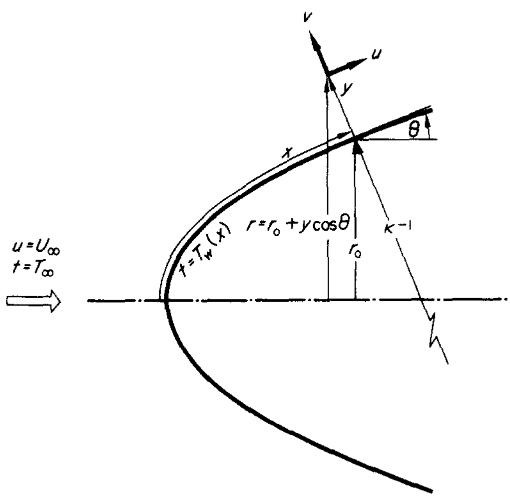

FIG. 1. Coordinate system.

length of the body L and velocities and temperatures to respective characteristic values, U_r and T_r. We assume for simplicity that the temperature far upstream, T_∞, is independent of the Péclet number. The governing equation for temperature is

$$\mathbf{q}\,.\,\mathrm{grad}\, t - Pe^{-1}\nabla^2 t = 0, \tag{1}$$

where $\mathbf{q} = (u, v, 0)$ is assumed to be known, t is the non-dimensional temperature referred to T_r and Pe the Péclet number ($= U_r L/\alpha$), α being the thermal diffusivity of the fluid. The boundary conditions are

$$\left.\begin{array}{lll} & t = T_w(x) & \text{at } y = 0 \\ \text{and} & t \to T_\infty & \text{upstream.}\end{array}\right\} \tag{2}$$

The formulation of the thermal boundary-layer theory can be made by using the method of matched asymptotic expansions with $Pe^{-\frac{1}{2}}$ as a perturbation parameter. According to this method, the temperature is obtained as an expansion in terms of $Pe^{-\frac{1}{2}}$ in each of two regions; one is the thermal boundary-layer region (inner region) adjacent to the wall with the thickness of the order $Pe^{-\frac{1}{2}}$, and the other is the region outside this layer (outer region).

In the outer region we assume an expansion of the form (outer expansion)

$$t = t_0(x, y) + Pe^{-\frac{1}{2}} t_1(x, y) + O(Pe^{-1}), \quad \text{as } Pe \to \infty \text{ with } x \text{ and } y \text{ fixed.} \tag{3}$$

Substituting (3) into (1) gives

$$\mathbf{q}\,.\,\mathrm{grad}\, t_0 = \mathbf{q}\,.\,\mathrm{grad}\, t_1 = 0. \tag{4}$$

This shows that t_0 and t_1 are constant along the streamline and are therefore functions only of the stream function ψ, which is related to u and v by the relation

$$\left.\begin{array}{ll} & \partial\psi/\partial y = r^j u \\ \text{and} & \partial\psi/\partial x = -r^j(1+\kappa y)v,\end{array}\right\} \tag{5}$$

with $j = 0$ for plane flow and $j = 1$ for axisymmetric flow. Hence, using the upstream condition and noting that T_∞ is independent of Pe, we have

$$t_0 = H(\psi), \tag{6}$$

$$t_1 = 0, \tag{7}$$

where $H(\psi)$ denotes a function only of ψ and are to be evaluated from the upstream condition. Since the order of equation (4) is reduced by one, the boundary condition on the surface has been dropped; therefore solutions (6) and (7) are invalid in the inner region.

In the inner region near the surface, we stretch the normal coordinate by introducing the following inner variable

$$Y = Pe^{\frac{1}{2}} y, \tag{8}$$

and assume an expansion of the form (inner expansion)

$$t = T_0(x, Y) + Pe^{-\frac{1}{2}} T_1(x, Y) + O(Pe^{-1}), \quad \text{as } Pe \to \infty \text{ with } x \text{ and } Y \text{ fixed.} \tag{9}$$

Since, in this region, $y \ll 1$, the longitudinal velocity u may be expressed as

$$u = u(x, 0) + (\partial u/\partial y)_{y=0}\, y + O(y^2),$$

which can be written as an expansion in terms of $Pe^{-\frac{1}{2}}$ [5]

$$u = u_p + Pe^{-\frac{1}{2}}\{B'(0) r_0^j - \kappa u_p\} Y + O(Pe^{-1}), \tag{10}$$

where $u_p = u(x, 0)$ and $B(\psi) = p + \frac{1}{2}|\mathbf{q}|^2$ (Bernoulli function), (11)

p being the non-dimensional pressure referred to ρU_r^2. The normal velocity v may be obtained by integrating the equation of continuity

$$\frac{\partial}{\partial x}(r^j u) + \frac{\partial}{\partial y}\{r^j(1+\kappa y)v\} = 0, \tag{12}$$

to yield

$$v = -Pe^{-\frac{1}{2}} r_0^{-j} \frac{\mathrm{d}(r_0^j u_p)}{\mathrm{d}x} Y + Pe^{-1} r_0^{-j}\left[-\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}x}\left\{r_0^j\left(B'(0) r_0^j - \kappa u_p + \frac{j\cos\theta}{r_0} u_p\right)\right\} + \left(\kappa + \frac{j\cos\theta}{r_0}\right)\frac{\mathrm{d}(r_0^j u_p)}{\mathrm{d}x}\right] Y^2 + O(Pe^{-\frac{3}{2}}). \tag{13}$$

Substituting (8), (9), (10) and (13) into (1) gives the following equations.

First-order equation:

$$u_p\frac{\partial T_0}{\partial x}-r_0^{-j}\frac{\mathrm{d}(r_0^j u_p)}{\mathrm{d}x}Y\frac{\partial T_0}{\partial Y}-\frac{\partial^2 T_0}{\partial Y^2}=0. \quad (14)$$

Second-order equation:

$$u_p\frac{\partial T_1}{\partial x}-r_0^{-j}\frac{\mathrm{d}(r_0^j u_p)}{\mathrm{d}x}Y\frac{\partial T_1}{\partial Y}-\frac{\partial^2 T_1}{\partial Y^2}$$
$$=\left(\kappa+\frac{j\cos\theta}{r_0}-\frac{Y^2}{r_0^j}\left[-\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}x}\left\{r_0^j\left(B'(0)r_0^j-\kappa u_p+\frac{j\cos\theta}{r_0}u_p\right)\right\}+\left(\kappa+\frac{j\cos\theta}{r_0}\right)\frac{\mathrm{d}(r_0^j u_p)}{\mathrm{d}x}\right]\right)\frac{\partial T_0}{\partial Y}$$
$$+\{2\kappa Y u_p-B'(0)r_0^j Y\}\frac{\partial T_0}{\partial x}. \quad (15)$$

The corresponding boundary conditions are

$$T_0(x,0)=T_w(x) \quad \text{and} \quad T_1(x,0)=0. \quad (16)$$

Since the inner expansion is invalid in the outer region, the upstream condition cannot be imposed on it. The missing condition is supplied by the matching condition that the outer expansion (3) and the inner expansion (9) must agree in the overlapping domain in which both expansions are valid. The matching condition can be written as [7]

$$\lim_{Y\to\infty}\{T_0(x,Y)+Pe^{-\frac{1}{2}}T_1(x,Y)+\ldots\}$$
$$=\lim_{y\to 0}\{t_0(x,y)+Pe^{-\frac{1}{2}}t_1(x,y)+\ldots\}. \quad (17)$$

Bearing in mind that t_0 and t_1 are given by (6) and (7) respectively, it can be easily shown that this matching condition reduces to

$$T_0(x,Y)\sim H(0) \quad (18)$$

and

$$T_1(x,Y)\sim r_0^j u_p H'(0)Y, \quad (19)$$

as $Y\to\infty$. Thus, it is now found that the present problem is to solve equations (14) and (15) under the conditions (16), (18) and (19).

It is advantageous, as for the case of viscous flow, to introduce the Görtler type variables defined by

$$\xi=\int_0^x u_p r_0^{2j}\,\mathrm{d}x \quad \text{and} \quad \zeta=\frac{r_0^j u_p}{\sqrt{(2\xi)}}Y. \quad (20)$$

The first- and the second-order temperature are also changed to the following forms,

$$\theta_0(\xi,\zeta)=\frac{T_0-H(0)}{T_w-H(0)} \quad \text{and} \quad \theta_1(\xi,\zeta)=\frac{T_1}{T_w-H(0)}. \quad (21)$$

Since the second-order equation (15) is linear, θ_1 may be divided into four components which represent direct contributions due to longitudinal curvature (θ_1^l), transverse curvature (θ_1^t), vorticity (θ_1^x) and external temperature gradient (θ_1^z), as

$$\theta_1=k_l\theta_1^l+k_t\theta_1^t+k_x\theta_1^x+k_z\theta_1^z, \quad (22)$$

where

$$k_l=(2\xi)^{\frac{1}{2}}\kappa/r_0^j u_p, \quad (23)$$
$$k_t=(2\xi)^{\frac{1}{2}}j\cos\theta/r_0^{2j}u_p, \quad (24)$$
$$k_x=(2\xi)^{\frac{1}{2}}B'(0)/u_p^2 \quad (25)$$

and

$$k_z=(2\xi)^{\frac{1}{2}}H'(0)/(T_w-H(0)). \quad (26)$$

Substituting (20)–(22) into (14)–(16) gives the following equations and boundary conditions.

First-order equation:

$$\theta_0''+\zeta\theta_0'-\gamma\theta_0=2\xi\frac{\partial\theta_0}{2\xi}, \qquad \theta_0(\xi,0)=1, \quad \theta_0(\xi,\infty)=0. \quad (27)$$

Second-order equations:

$$\theta_1^{l''}+\zeta\theta_1^{l'}-(\gamma+\pi_l)\theta_1^l=2\xi\left(\frac{\partial\theta_1^l}{\partial\xi}-2\zeta\frac{\partial\theta_0}{\partial\xi}\right)+\left(\frac{\pi_l+3}{2}\zeta^2-1\right)\theta_0'-2\gamma\zeta\theta_0,$$
$$\theta_1^l(\xi,0)=\theta_1^l(\xi,\infty)=0, \quad (28)$$

$$\theta_1^{t''}+\zeta\theta_1^{t'}-(\gamma+\pi_t)\theta_1^t=2\xi\frac{\partial\theta_1^t}{\partial\xi}+\left(\frac{1-\pi_t}{2}\zeta^2-1\right)\theta_0',$$
$$\theta_1^t(\xi,0)=\theta_1^t(\xi,\infty)=0, \quad (29)$$

$$\theta_1^{x''}+\zeta\theta_1^{x'}-(\gamma+\pi_x)\theta_1^x=2\xi\left(\frac{\partial\theta_1^x}{\partial\xi}+\zeta\frac{\partial\theta_0}{\partial\xi}\right)-\frac{\pi_x+1}{2}\zeta^2\theta_0'+\gamma\zeta\theta_0,$$
$$\theta_1^x(\xi,0)=\theta_1^x(\xi,\infty)=0, \quad (30)$$

$$\theta_1^{z''}+\zeta\theta_1^{z'}-\theta_1^z=2\xi\frac{\partial\theta_1^z}{\partial\xi}, \quad \theta_1^z(\xi,0)=0, \quad \theta_1^z(\xi,\infty)=\zeta. \quad (31)$$

Here

$$\left.\begin{aligned}\pi_i&=\frac{2\xi}{k_i}\frac{\mathrm{d}k_i}{\mathrm{d}\xi} \qquad (i=l,t,x),\\ \gamma&=\frac{2\xi}{T_w-H(0)}\frac{\mathrm{d}(T_w-H(0))}{\mathrm{d}\xi}.\end{aligned}\right\} \quad (32)$$

and primes denote the differentiation with respect to ζ. The principal functions π_i's and γ are, in general, functions of ξ. If the principal thermal function γ remains constant in stream direction, the first-order temperature, θ_0, is similar (that is, θ_0 is a function only of ζ) and, if both γ and π_i remain constant, θ_1^i is also similar. Moreover, it is seen from (31) that the equation for θ_1^z involves no principal function, meaning that θ_1^z is always similar.

Finally, the local Nusselt number defined by

$$Nu=\frac{qL}{kT_r(T_w-H(0))},$$

where q is the heat flux at the wall, is expressed as

$$Nu/Pe^{\frac{1}{2}}=-\frac{r_0^j u_p}{(2\xi)^{\frac{1}{2}}}[\theta_0'+Pe^{-\frac{1}{2}}(k_l\theta_1^{l'}+k_t\theta_1^{t'}+k_x\theta_1^{x'}+k_z\theta_1^{z'})+O(Pe^{-1})]_{\zeta=0}. \quad (33)$$

3. FIRST-ORDER SOLUTION

The exact solution of the first-order equation (27) has already been obtained by Morgan and Warner [8]. In the solution, however, an integral which, in general, has to be evaluated numerically is contained. Since this is not desirable for practical use, we here seek an approximate solution which permits rapid calculation of heat transfer. For viscous flow, a large variety of the approximate methods for calculating heat transfer through a non-similar boundary layer have been proposed so far by many authors [9] and several authors [9–11] claimed that one of the most satisfactory methods is an approximate method due to Merk [6]. The principal idea of the method is to expand asymptotically the full non-similar boundary-layer equations in terms of a small parameter which is a measure of the departure of the solutions from similarity, first terms of the expansions corresponding to so-called local similarity solutions. Great advantage of the method is that it is possible to refine the results in a straightforward way by calculating the higher order terms. Moreover, it provides one of the most rapid calculations hitherto propounded.

In the present paper, the Merk's method is extended to obtaining non-similar solutions of the equations (27)–(30). The key to an appropriate expansion is the inversion of an independent variable ξ. Since γ is a given function of ξ for a given shape of the body and given temperature distribution on the wall, we may also say, inverting this function, that ξ is a function of γ. Thus, equation (27) may be written as

$$\theta_0''+\zeta\theta_0'-\gamma\theta_0 = \varepsilon(\gamma)\frac{\partial\theta_0}{\partial\gamma}, \tag{34}$$

where

$$\varepsilon(\gamma) = 2\xi\frac{\mathrm{d}\gamma}{\mathrm{d}\xi}, \tag{35}$$

and θ_0, assuming ε is small, as

$$\theta_0 = \theta_{00}(\gamma,\zeta)+\varepsilon(\gamma)\theta_{01}(\gamma,\zeta)+O(\varepsilon^2). \tag{36}$$

Substituting (36) into (34) gives

$$\theta_{00}''+\zeta\theta_{00}'-\gamma\theta_{00} = 0, \tag{37}$$

$$\theta_{01}''+\zeta\theta_{01}'-(\gamma+\varepsilon')\theta_{01} = \frac{\partial\theta_{00}}{\partial\gamma}, \tag{38}$$

where ε' denotes $\mathrm{d}\varepsilon/\mathrm{d}\gamma$. For the special case where γ remains constant in stream direction, $\varepsilon(\gamma) = 0$ and therefore equation (37) yields an exact similar solution. Equation (38) represents the first correction which arises from non-similar terms.

The solutions of the above equations satisfying the respective boundary conditions and matching conditions are easily found to be

$$\theta_{00} = \frac{2^{\gamma/2}\Gamma\left(\frac{\gamma+2}{2}\right)}{(\pi/2)^{\frac{1}{2}}}\bar{V}(\gamma,\zeta) \tag{39}$$

and

$$\theta_{01} = \begin{cases} -\dfrac{1}{\varepsilon'^2}\theta_{00}-\dfrac{1}{\varepsilon'}\dfrac{\partial\theta_{00}}{\partial\gamma}+\dfrac{1}{\varepsilon'^2}\dfrac{2^{(\gamma+\varepsilon')/2}\Gamma\left(\frac{\gamma+\varepsilon'+2}{2}\right)}{(\pi/2)^{\frac{1}{2}}} \\ \qquad \times\ \bar{V}(\gamma+\varepsilon',\zeta) \quad \text{for} \quad \varepsilon' \neq 0, & (40) \\ \frac{1}{2}\dfrac{\partial^2\theta_{00}}{\partial\gamma^2} \quad \text{for} \quad \varepsilon' = 0, & (41) \end{cases}$$

where

$$\bar{V}(\gamma,\zeta) = \frac{(\pi/2)^{\frac{1}{2}}}{2^{\gamma/2}\Gamma\left(\frac{\gamma+2}{2}\right)}\,{}_1F_1\left(-\frac{\gamma}{2},\frac{1}{2},\frac{\zeta^2}{2}\right) - \frac{\zeta\sqrt{\pi}}{2^{\gamma/2}\Gamma\left(\frac{\gamma+1}{2}\right)}\,{}_1F_1\left(-\frac{\gamma-1}{2},\frac{3}{2},\frac{\zeta^2}{2}\right), \tag{42}$$

${}_1F_1(a,b,x)$ being the confluent hypergeometric function satisfying the equation

$$x\frac{\mathrm{d}^2u}{\mathrm{d}x^2}+(b-x)\frac{\mathrm{d}u}{\mathrm{d}x}-au = 0. \tag{43}$$

From the above solutions, the temperature gradient at the surface can be obtained as

$$-(\theta_0')_{\zeta=0} = -\theta_{00}'(\gamma,0)-\varepsilon(\gamma)\theta_{01}'(\gamma,0)+O(\varepsilon^2) \tag{44}$$

$$= \frac{\Gamma\left(\frac{\gamma+2}{2}\right)\sqrt{2}}{\Gamma\left(\frac{\gamma+1}{2}\right)}\left\{1+\varepsilon(\gamma)\frac{\theta_{01}'(\gamma,0)}{\theta_{00}'(\gamma,0)}+O(\varepsilon^2)\right\}, \tag{45}$$

where

$$\frac{\theta_{01}'(\gamma,0)}{\theta_{00}'(\gamma,0)} = \begin{cases} \dfrac{1}{\varepsilon'}\left[\dfrac{\Psi\left(\frac{\gamma+1}{2}\right)-\Psi\left(\frac{\gamma+2}{2}\right)}{2} + \dfrac{1}{\varepsilon'}\left\{\dfrac{\Gamma\left(\frac{\gamma+1}{2}\right)\Gamma\left(\frac{\gamma+\varepsilon'+2}{2}\right)}{\Gamma\left(\frac{\gamma+2}{2}\right)\Gamma\left(\frac{\gamma+\varepsilon'+1}{2}\right)}-1\right\}\right] \\ \qquad \text{for} \quad \varepsilon' \neq 0, & (46) \\ \frac{1}{8}\left[\left\{\Psi\left(\frac{\gamma+2}{2}\right)-\Psi\left(\frac{\gamma+1}{2}\right)\right\}^2 + \Psi'\left(\frac{\gamma+2}{2}\right)-\Psi'\left(\frac{\gamma+1}{2}\right)\right] \\ \qquad \text{for} \quad \varepsilon' = 0, & (47) \end{cases}$$

Ψ and Ψ' being the polygamma functions defined by

$$\Psi(z) = \Gamma'(z)/\Gamma(z), \qquad \Psi'(z) = \mathrm{d}\Psi(z)/\mathrm{d}z.$$

It is seen from the above expressions that for $\gamma < 0$ these heat-transfer results have an infinite number of singularities due to Gamma or polygamma functions, the first of these singularities being at $\gamma = -2$ for $\theta_{00}'(\gamma,0)$ and at $\gamma = -1$ for $\theta_{01}'(\gamma,0)$. Similar singularities are observed also in the second-order solutions, as we shall see later.

Figure 2 shows the curves of $\theta_{00}'(\gamma,0)$ and $\theta_{01}'(\gamma,0)$ plotted as functions of γ. It is seen from the figures that the ratio $|\theta_{01}'(\gamma,0)/\theta_{00}'(\gamma,0)|$ is very small for $\gamma > 0$; this demonstrates the rapid convergence of the series (44) in this range of γ. For $\gamma < 0$, on the other hand, the ratio becomes larger as γ approaches nearer to -1.

For a given body and given temperature distribution, γ, ε and ε' are known functions of x, so that, for each value of x, the corresponding value of $(\theta_0')_{\zeta=0}$ can be calculated from equations (44)–(47). As an example, we shall calculate heat transfer from an elliptical rod located in the interior of an elliptical-rod bundle with square spacing or equilateral triangular spacing, such as those shown in Fig. 3. This problem was already treated by Hsu [12], but only for the special case when the wall temperature distribution is given by

$$T_w - H(0) = (1-\cos\eta)^m \quad \text{with} \quad m = 0 \text{ and } 1,$$

η being the angle measured from the front stagnation point (Fig. 4). Hsu made the following assumptions: (1) flow is irrotational; (2) the distribution of hydrodynamic potential around the surface of the rod is of the cosine type in terms of elliptical cylindrical coordinates; and (3) interaction of the thermal boundary layers of the cylinders in a rod bundle is negligible.

In the following, we will also make the same assumptions as the above. Then, defining the reference velocity by $U_r = U_\infty$ and reference length by $L = 2a$, respectively, u_p can be written as

$$u_p^2 = \frac{\phi_1^2(1-\cos 2\eta)}{4b^2\left\{1+\dfrac{a^2}{b^2}+\left(1-\dfrac{a^2}{b^2}\right)\cos 2\eta\right\}}, \tag{48}$$

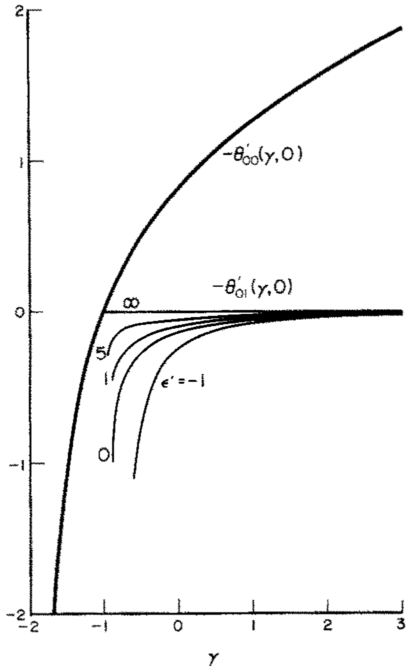

FIG. 2. First-order solution.

where $\phi_1 = (\Phi_1 - \Phi_2)/U_\infty$ is given in tabular form by Hsu [12, 13] as a function of $(a+b)/2P$, Φ_1 and Φ_2 being the values of velocity potential at the front and rear stagnation points of the cylinder, and ξ as

$$\xi = (\phi_1/4b)(1-\cos\eta). \tag{49}$$

Substituting (45), (48) and (49) into (33) gives for the local Nusselt number

$$\frac{Nu}{Pe^{\frac{1}{2}}} = \left(\frac{\phi_1}{a+b}\right)^{\frac{1}{2}}\left[\frac{\{(1-e^2)+\sqrt{(1-e^2)}\}(1+\cos\eta)}{1-e^2\cos^2\eta}\right]^{\frac{1}{2}} \times \frac{\Gamma\left(\dfrac{\gamma+2}{2}\right)}{\Gamma\left(\dfrac{\gamma+1}{2}\right)}\left\{1+\varepsilon\left(\frac{\theta'_{01}}{\theta'_{00}}\right)_{\zeta=0}+O(\varepsilon^2)\right\}, \tag{50}$$

where $e = (1-b^2/a^2)$. It can be easily verified that when the temperature distribution of the surface is given by

$$T_w - H(0) = (1-\cos\eta)^m \quad \text{with} \quad m = \text{constant}, \tag{51}$$

γ is constant $(=2m)$ and therefore the local Nusselt number is exactly expressed by the first term in (50), which includes the results of Hsu as special cases. As an example of non-similar case, we calculated heat

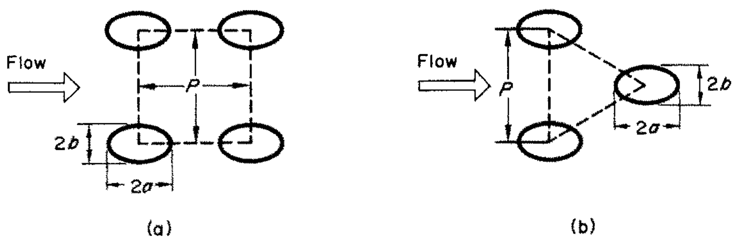

FIG. 3. Schematic representation of tube bank geometries: (a) square spacing, (b) equilateral triangular spacing.

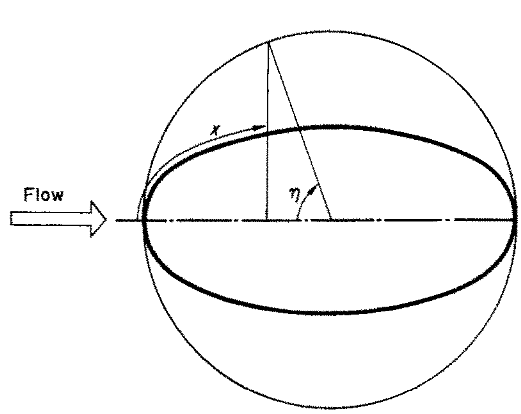

FIG. 4. Angle, η, measured from the front stagnation point.

transfer when the wall temperature distribution is given by

$$T_w - H(0) = 2-\cos\eta, \tag{52}$$

for which an exact solution can be obtained by superposition of similar solutions. The heat-transfer result is shown in Fig. 5; the present result with two terms

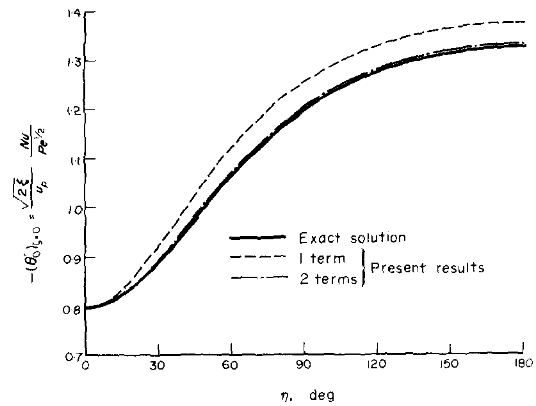

FIG. 5. Heat-transfer results for tube banks.

agree with the exact one with error within about 0·4 per cent and, if the desired accuracy is not higher than 5 per cent, one term is sufficient to calculate heat transfer.

4. SECOND-ORDER SOLUTIONS

As is stated before, the equation (31) governing the second-order effect of external temperature gradient has a similar solution, which can be easily found to be

$$\theta_1^z = \zeta. \tag{53}$$

From this, the wall temperature gradient can be expressed as

$$(\theta_1^{z\prime})_{\zeta=0} = 1. \tag{54}$$

The sign of this second-order contribution to heat transfer is opposite to that of the first-order contributions $(\theta_0')_{\zeta=0}$, except for a special case in which a heat flow is directed from the stream to the wall even if the local wall temperature is greater than the local stream temperature, as is often the case with the situation in which γ is negative; hence it can be said except for this special case that when $k_z > 0$ the presence of the external temperature gradient decreases heat transfer, while when $k_z < 0$ it increases heat transfer. For the special case stated above, on the other hand, this trend is reversed. It can be easily understood that these results are physically reasonable.

Since the equations governing the remaining three second-order effects have no such similar solutions as the above, we shall employ the approximate procedure similar to that used in the first-order problem. Each of the three equations has two principal functions, γ and π_i, which are functions of ξ; therefore even for the special case in which γ is constant in stream direction, the solutions of the equations are, in general, not similar. It is clear that for this special case γ cannot be used as an independent variable in applying Merk's method, and that π_i should be used instead. [But, as we shall see later, the equation for θ_1^l has a similar solution when γ is constant (but, π_l is not, in general, constant) and therefore Merk's procedure is not needed.] For the general case in which both γ and π_i vary in stream direction, on the other hand, γ can be used as an independent variable as for the case of the first-order problem.

(a) *The case of* γ = *constant*

We first consider the simpler case γ = constant. For this special case, the first-order temperature is exactly expressed by θ_{00}, which is given by (39). Hence, equations (28)–(30) may be written as

$$\theta_1^{l\prime\prime}+\zeta\theta_1^{l\prime}-(\gamma+\pi_l)\theta_1^l = \frac{2^{\gamma/2}\Gamma\left(\frac{\gamma+2}{2}\right)}{(\pi/2)^{\frac{1}{2}}}\left\{\frac{\gamma(\gamma+1)(1-\pi_l)}{2}\bar{V}(\gamma+1,\zeta) + \frac{(2\gamma-1)(\pi_l+1)}{2}\bar{V}(\gamma-1,\zeta) - \frac{\pi_l+3}{2}\bar{V}(\gamma-3,\zeta)\right\}+2\xi\frac{\partial\theta_1^l}{\partial\xi}, \tag{55}$$

$$\theta_1^{t\prime\prime}+\zeta\theta_1^{t\prime}-(\gamma+\pi_t)\theta_1^t = \frac{2^{\gamma/2}\Gamma\left(\frac{\gamma+2}{2}\right)}{(\pi/2)^{\frac{1}{2}}}\left\{\frac{\gamma(\gamma+1)(\pi_t-1)}{2}\bar{V}(\gamma+1,\zeta) + \frac{(2\gamma-1)(1-\pi_t)+2}{2}\bar{V}(\gamma-1,\zeta) - \frac{1-\pi_t}{2}\bar{V}(\gamma-3,\zeta)\right\}+2\xi\frac{\partial\theta_1^t}{\partial\xi}, \tag{56}$$

$$\theta_1^{x\prime\prime}+\zeta\theta_1^{x\prime}-(\gamma+\pi_x)\theta_1^x = \frac{2^{\gamma/2}\Gamma\left(\frac{\gamma+2}{2}\right)}{(\pi/2)^{\frac{1}{2}}}\left\{\frac{\gamma(\gamma+1)(\pi_x-1)}{2}\bar{V}(\gamma+1,\zeta) + \frac{(1-2\gamma)\pi_x+1}{2}\bar{V}(\gamma-1,\zeta) + \frac{1+\pi_x}{2}\bar{V}(\gamma-3,\zeta)\right\}+2\xi\frac{\partial\theta_1^x}{\partial\xi}. \tag{57}$$

We can easily find that (55) has a following similar solution

$$\theta_1^l = \frac{2^{\gamma/2}\Gamma\left(\frac{\gamma+2}{2}\right)}{(\pi/2)^{\frac{1}{2}}}\left\{\frac{\gamma(\gamma+1)}{2}\bar{V}(\gamma+1,\zeta) - \frac{2\gamma-1}{2}\bar{V}(\gamma-1,\zeta)+\tfrac{1}{2}\bar{V}(\gamma-3,\zeta)\right\}. \tag{58}$$

Thus, the temperature profile representing the second-order effect of longitudinal curvature is independent of the principal function π_l. Furthermore, we can obtain, from (58), the following interesting result

$$(\theta_1^{l\prime})_{\zeta=0} = 0, \tag{59}$$

meaning that when γ = const., the longitudinal curvature does not contribute to heat transfer to the present order of Péclet number. For viscous flow, Afzal and Oberai [14] showed that the convex longitudinal surface curvature ($k_l > 0$) decreases heat transfer, and that this decrease in heat transfer is smaller for smaller values of Pr. Since inviscid flow approximation corresponds to the case $Pr \to 0$, the above result (59) is consistent with this tendency. According to Hirose [15], the convex curvature effects consist of two contributions: (a) increase of the diffusion area and (b) decrease of tangential velocity with increasing normal distance from the surface of the body. The former enhances the transfer rate while the latter lowers it. Therefore, Afzal's result together with the present result (59) indicates that for viscous flow the latter effect is of major importance, but that with decrease of Pr the former effect increases and, in the limit $Pr \to 0$, i.e. for the case of inviscid flow, the magnitude of both effects becomes to be equal with opposite sign. The physical meaning of this trend is explained as follows: for low Prandtl number fluids, heat conduction plays a more important part in heat transfer and therefore increase of the diffusion area is more effective for increasing heat transfer than for higher Prandtl number fluids.

Since the equations for θ_1^t and θ_1^x have no such similar solutions as (58), we must employ, for solving them, the method similar to that used in the first-order problem; that is, we change variables from (ξ, ζ) to (π_i, ζ) with $i = t$ or x and assume the following expansion.

$$\theta_1^i = \theta_{10}^i(\pi_i, \zeta) + \varepsilon_i(\pi_i)\theta_{11}^i(\pi_i, \zeta) + O(\varepsilon_i^2), \tag{60}$$

where

$$\varepsilon_i = 2\xi \frac{d\pi_i}{d\xi}. \tag{61}$$

Then, we have the following equations for θ_{10}^i and θ_{11}^i,

$$\theta_{10}^{i''} + \zeta\theta_{10}^{i'} - (\gamma+\pi_i)\theta_{10}^i = \frac{2^{\gamma/2}\Gamma\left(\frac{\gamma+2}{2}\right)}{(\pi/2)^{\frac{1}{2}}}\left\{\frac{\gamma(\gamma+1)(\pi_i-1)}{2}\bar{V}(\gamma+1,\zeta) + \frac{\pi_i(1-2\gamma)+a}{2}\bar{V}(\gamma-1,\zeta) + \frac{\pi_i - b}{2}\bar{V}(\gamma-3,\zeta)\right\}, \tag{62}$$

$$\theta_{11}^{i''} + \zeta\theta_{11}^{i'} - (\gamma+\pi_i+\varepsilon_i')\theta_{11}^i = \frac{\partial\theta_{10}^i}{\partial\pi_i}, \tag{63}$$

where a and b are given in Table 1 and ε_i' denotes $d\varepsilon_i/d\pi_i$. The solutions of (62) and (63) satisfying respective boundary conditions are

$$\theta_{10}^i = \frac{2^{\gamma/2}\Gamma\left(\frac{\gamma+2}{2}\right)}{(\pi/2)^{\frac{1}{2}}}\left\{-\frac{\gamma(\gamma+1)}{2}\bar{V}(\gamma+1,\zeta) - \frac{\pi_i(1-2\gamma)+a}{2(\pi_i+1)}\bar{V}(\gamma-1,\zeta) - \frac{\pi_i-b}{2(\pi_i+3)}\bar{V}(\gamma-3,\zeta) + \frac{2^{(\pi_i+1)/2}(\pi_i+2\gamma+1)\,c\Gamma\left(\frac{\gamma+\pi_i+2}{2}\right)}{(\gamma-1)(\pi_i+1)(\pi_i+3)\Gamma\left(\frac{\gamma-1}{2}\right)}\right\}, \tag{64}$$

$$\theta_{11}^i = \begin{cases} -\dfrac{1}{\varepsilon_i'^2}\theta_{10}^i - \dfrac{1}{\varepsilon_i'}\dfrac{\partial\theta_{10}^i}{\partial\pi_i} - \dfrac{2^{\gamma/2}\Gamma\left(\frac{\gamma+2}{2}\right)}{2\varepsilon_i'^2(\pi/2)^{\frac{1}{2}}}\Bigg\{\gamma(\gamma+1)\bar{V}(\gamma+1,\zeta) - \left(2\gamma-1+\dfrac{1-2\gamma-a}{\pi_i+\varepsilon_i'+1}\right)\bar{V}(\gamma-1,\zeta) \\ \quad - \left(\dfrac{b+3}{\pi_i+\varepsilon_i'+3}-1\right)\bar{V}(\gamma-3,\zeta) - \dfrac{2^{\frac{\pi_i+\varepsilon_i'+1}{2}}(2\gamma+\pi_i+\varepsilon_i'+1)c\Gamma\left(\frac{\gamma+\pi_i+\varepsilon_i'+2}{2}\right)}{(\pi_i+\varepsilon_i'+1)(\pi_i+\varepsilon_i'+3)\Gamma\left(\frac{\gamma+1}{2}\right)} \\ \quad \times \bar{V}(\gamma+\pi_i+\varepsilon_i',\zeta)\Bigg\} \quad \text{for} \quad \varepsilon_i' \neq 0, & (65) \\ \dfrac{1}{2}\dfrac{\partial^2\theta_{10}^i}{\partial\pi_i^2} \quad \text{for} \quad \varepsilon_i' = 0, & (66) \end{cases}$$

where c is given in Table 1. From these solutions, the temperature gradient at the surface can be obtained as

$$-(\theta_{10}^{i'})_{\zeta=0} = \frac{c}{(\pi_i+3)(\pi_i+1)}\left\{-\gamma(\gamma+\pi_i+1) + \frac{(2\gamma+\pi_i+1)\Gamma\left(\frac{\gamma+2}{2}\right)\Gamma\left(\frac{\gamma+\pi_i+2}{2}\right)}{\Gamma\left(\frac{\gamma+1}{2}\right)\Gamma\left(\frac{\gamma+\pi_i+1}{2}\right)}\right\}, \tag{67}$$

$$-(\theta_{11}^{i'})_{\zeta=0} = \begin{cases} \dfrac{1}{\varepsilon_i'^2}\theta_{10}^{i'}(\pi_i,0) + \dfrac{1}{\varepsilon_i'}\dfrac{d}{d\pi_i}\{\theta_{10}^{i'}(\pi_i,0)\} \\ \quad + \dfrac{c}{(\pi_i+\varepsilon_i'+1)(\pi_i+\varepsilon_i'+3)\varepsilon_i'^2}\left\{\dfrac{(2\gamma+\pi_i+\varepsilon_i'+1)\Gamma\left(\frac{\gamma+2}{2}\right)\Gamma\left(\frac{\gamma+\pi_i+\varepsilon_i'+2}{2}\right)}{\Gamma\left(\frac{\gamma+1}{2}\right)\Gamma\left(\frac{\gamma+\pi_i+\varepsilon_i'+1}{2}\right)} - \gamma(\gamma+\pi_i+\varepsilon_i'+1)\right\} \\ \quad \text{for} \quad \varepsilon_i' \neq 0, & (68) \\ \dfrac{2(\pi_i+2)}{(\pi_i+1)(\pi_i+3)}\dfrac{d}{d\pi_i}\{\theta_{10}^{i'}(\pi_i,0)\} + \dfrac{1}{(\pi_i+1)(\pi_i+3)}\theta_{10}^{i'}(\pi_i,0) \\ \quad + \dfrac{c\Gamma\left(\frac{\gamma+2}{2}\right)\Gamma\left(\frac{\gamma+\pi_i+2}{2}\right)}{2(\pi_i+1)(\pi_i+3)\Gamma\left(\frac{\gamma+1}{2}\right)\Gamma\left(\frac{\gamma+\pi_i+1}{2}\right)}\Bigg[\Psi\left(\frac{\gamma+\pi_i+2}{2}\right) - \Psi\left(\frac{\gamma+\pi_i+1}{2}\right) \\ \quad + \dfrac{2\gamma+\pi_i+1}{4}\left\{\left(\Psi\left(\frac{\gamma+\pi_i+2}{2}\right) - \Psi\left(\frac{\gamma+\pi_i+1}{2}\right)\right)^2 + \Psi'\left(\frac{\gamma+\pi_i+2}{2}\right) - \Psi'\left(\frac{\gamma+\pi_i+1}{2}\right)\right\}\Bigg] \\ \quad \text{for} \quad \varepsilon_i' = 0, & (69) \end{cases}$$

where

$$\frac{d}{d\pi_i}\{\theta_{10}^{i'}(\pi_i,0)\} = -\frac{2(\pi_i+2)}{(\pi_i+1)(\pi_i+3)}\theta_{10}^{i'}(\pi_i,0)+\frac{c}{(\pi_i+1)(\pi_i+3)} \times\left[\gamma-\frac{\Gamma\left(\frac{\gamma+2}{2}\right)\Gamma\left(\frac{\gamma+\pi_i+2}{2}\right)}{2\Gamma\left(\frac{\gamma+1}{2}\right)\Gamma\left(\frac{\gamma+\pi_i+1}{2}\right)}\left\{2+(2\gamma+\pi_i+1)\left(\Psi\left(\frac{\gamma+\pi_i+2}{2}\right)-\Psi\left(\frac{\gamma+\pi_i+1}{2}\right)\right)\right\}\right] \tag{70}$$

Table 1

i	a	b	c
t	$2\gamma+1$	1	4
x	1	-1	2

It is seen that these results have an infinite number of singularities when $\pi_i \leqslant -(\gamma+2)(\equiv \pi_{i,\mathrm{crit.}})$. Furthermore, we can obtain from these equations the following asymptotic relations;

$$\left.\begin{aligned}
&\lim_{\gamma\to\infty}[-(\theta_{10}^{i'})_{\zeta=0}] = c/8,\\
&\lim_{\pi_i\to\infty}[-(\theta_{10}^{i'})_{\zeta=0}] = 0,\\
&\lim_{\gamma\to\infty}[-(\theta_{11}^{i'})_{\zeta=0}] = \lim_{\pi_i\to\infty}[-(\theta_{11}^{i'})_{\zeta=0}]\\
&\qquad = \lim_{\epsilon_i'\to\infty}[-(\theta_{11}^{i'})_{\zeta=0}] = 0.
\end{aligned}\right\} \tag{71}$$

Figures 6 and 7 show the curves of $(\theta_{10}^{i'})_{\zeta=0}$ and $(\theta_{11}^{i'})_{\zeta=0}$, respectively, plotted as functions of π_i with γ as a parameter. It is seen from the figures that except

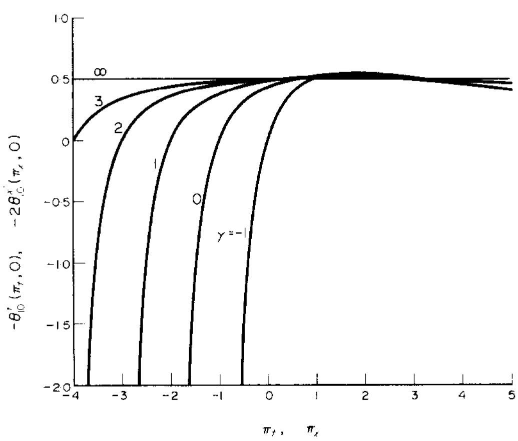

FIG. 6. The wall gradients, $\theta_{10}^{t'}(\pi_t, 0)$ and $\theta_{10}^{x'}(\pi_x, 0)$.

near $\pi_i = \pi_{i,\mathrm{crit.}}$, the ratio $|\theta_{10}^{i'}/\theta_{11}^{i'}|_{\zeta=0}$ is again very small. When $\gamma > -1$, sign of $(\theta_{10}^{i'})_{\zeta=0}$ is the same as that of the first-order contribution $(\theta_{00}')_{\zeta=0}$ for $\pi_i > \pi_{i,\mathrm{crit.}}+1$. Since k_t is always positive, this means that the transverse curvature increases heat transfer for this range of π_i. This increase in heat transfer is due to increase of the diffusion area. The presence of vorticity, on the other hand, increases heat transfer for $k_x > 0$ while for $k_x < 0$ it decreases heat transfer, provided $\pi_i > \pi_{i,\mathrm{crit.}}+1$.

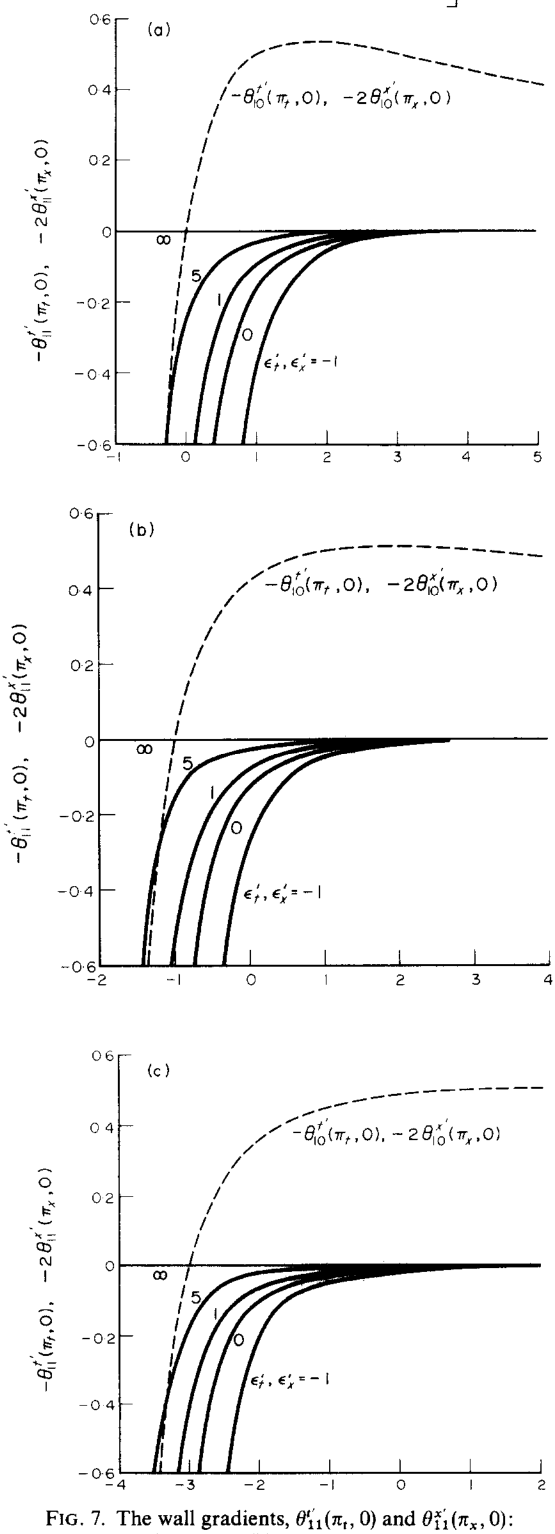

FIG. 7. The wall gradients, $\theta_{11}^{t'}(\pi_t, 0)$ and $\theta_{11}^{x'}(\pi_x, 0)$: (a) $\gamma = -1$, (b) $\gamma = 0$, (c) $\gamma = 2$.

Calculation of $(\theta_1^{t\prime})_{\zeta=0}$ for a given body and given temperature distribution can be made similarly to that of $(\theta_0')_{\zeta=0}$. As an example, we shall calculate heat transfer from an isothermal sphere, for which Watts [15] already calculated exactly the second-order effect of transverse curvature. Here we reconsider this problem including the effect of external temperature gradient also, but the velocity field is assumed, for simplicity, to be irrotational. Then, defining the reference length and reference velocity by $L = 2a$ and $U_r = U_\infty$, respectively, a being the radius of the sphere and U_∞ the uniform velocity of oncoming stream, we can finally obtain the following expression for the local Nusselt number,

$$Nu/Pe^{\frac{1}{2}} = \frac{3}{\pi^{\frac{1}{2}}}\frac{\sin^2\varphi}{(\cos^2\varphi - 3\cos\varphi + 2)^{\frac{1}{2}}} + Pe^{-\frac{1}{2}}\left\{-2(\theta_{10}^{l\prime} + \varepsilon_t\theta_{11}^{t\prime} + \dots)_{\zeta=0} - \tfrac{3}{4}\frac{H'(0)}{T_w - H(0)}\sin^2\varphi\right\} + O(Pe^{-1}), \quad (72)$$

where φ is the angle measured from the stagnation point of the sphere. Figure 8 shows the distribution of the wall gradient of the temperature representing the effect of transverse curvature. Figure 9 shows the distributions of the local Nusselt number around a sphere for $H'(0) = 0$. The average Nusselt number is obtained from the relation

$$\overline{Nu} = \tfrac{1}{2}\int_0^\pi Nu(\varphi)\sin\varphi\,d\varphi, \quad (73)$$

as

$$\overline{Nu}/Pe^{\frac{1}{2}} = \frac{2}{\pi^{\frac{1}{2}}} + Pe^{-\frac{1}{2}}\left\{(0{\cdot}85093 - 0{\cdot}00200 + \dots) - \tfrac{1}{2}\frac{H'(0)}{T_w - H(0)}\right\} + O(Pe^{-1})$$

$$= \frac{2}{\pi^{\frac{1}{2}}} + Pe^{-\frac{1}{2}}\left(0{\cdot}849 - \tfrac{1}{2}\frac{H'(0)}{T_w - H(0)}\right) + O(Pe^{-1}). \quad (74)$$

The numerical value 0·849 representing the effect of the transverse curvature is within about 3·4 per cent of the value obtained from the exact solution, 0·821.*

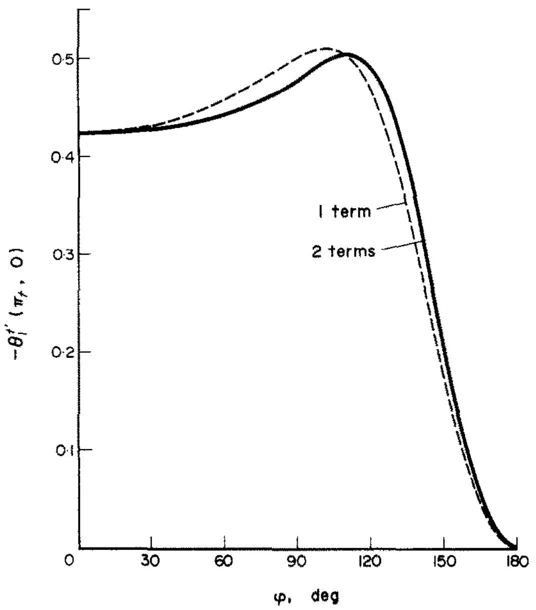

FIG. 8. The effect of transverse curvature on heat transfer from an isothermal sphere.

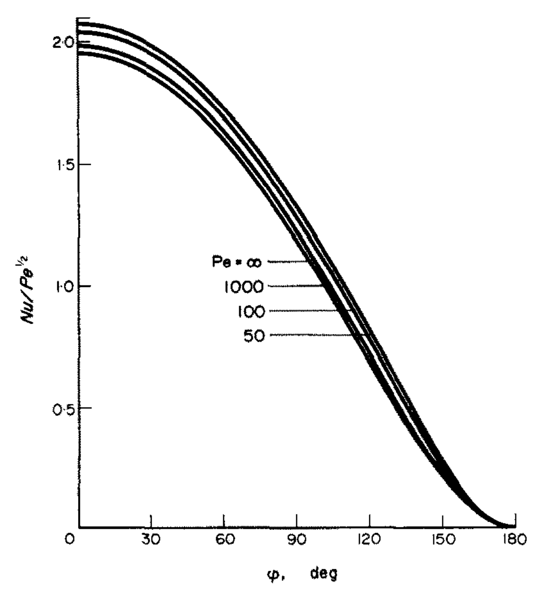

FIG. 9. Local Nusselt number around an isothermal sphere for $H'(0) = 0$.

(b) *The case of $\gamma \neq$ constant*

In this case, we change variables from (ξ, ζ) to (γ, ζ) as in the case of the first-order problem and assume the following expansion for θ_1^i's ($i = l, t, x$),

$$\theta_1^i = \theta_{10}^i(\gamma, \zeta) + \varepsilon(\gamma)\theta_{11}^i(\gamma, \zeta) + \dots, \quad (75)$$

where ε is defined by (35). By substituting this into (28)–(30), we can find that the equations for $\theta_{10}^i(\gamma, \zeta)$'s have the same forms as those for the previous case of γ = constant, so that the solutions given in the previous section, (58) and (64), can be immediately used also in the present case. The equations for θ_{11}^i's, on the other hand, are rather complicated and their solutions are not obtained here. However, judging from the accuracy of the present method shown in the previously given examples, we can expect that, if the desired accuracy for heat transfer is not higher than a few percent, one term in the series (75) will be sufficient to give satisfactory results.

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*Watts first derived the following equation,

$$\overline{Nu} = 1{\cdot}156Pe^{\frac{1}{2}} + 4{\cdot}73,$$

but which is found, by Hirose, to be in error and corrected as

$$\overline{Nu} = \frac{2}{\pi^{\frac{1}{2}}}Pe^{\frac{1}{2}} + 0{\cdot}821.$$

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APPROXIMATIONS D'ORDRE PLUS ELEVE DE LA COUCHE LIMITE THERMIQUE EN ECOULEMENT NON VISQUEUX

Résumé—Le problème du second ordre de la couche limite thermique est formulé pour un écoulement non visqueux plan ou axisymétrique. Les équations résultantes d'énergie qui contrôlent les effets des courbures longitudinale et transversale, du gradient de température et de la vorticité du courant extérieur, sont résolues de façon approchée et de façon exacte. Des applications de la théorie sont présentées pour le transfert de chaleur sur des grappes de tubes elliptiques et le transfert sur une sphère.

HÖHERE NÄHERUNGSVERFAHREN FÜR DIE THERMISCHE GRENZSCHICHT IN EINER NICHT VISKOSEN STRÖMUNG

Zusammenfassung—Ein thermisches Grenzschichtproblem 2. Ordnung wird für eine reibungsfreie ebene oder achsensymmetrische Strömung formuliert. Mit den erhaltenen Energiegleichungen lassen sich äußerer Temperaturgradient und Rotation sowohl exakt als auch angenähert bestimmen. Anwendungsbeispiele für die vorliegende Theorie auf die Wärmeübertragung von elliptischen Stabbündeln und von einer Kugel werden angegeben.

РЕШЕНИЕ ЗАДАЧИ ТЕПЛОВОГО СЛОЯ В ПРИБЛИЖЕНИИ БОЛЕЕ ВЫСОКОГО ПОРЯДКА ДЛЯ НЕВЯЗКОГО ПОТОКА

Аннотация — Формулируется задача теплового пограничного слоя для невязкого плоского и осесимметричного потока. Полученные уравнения энергии, описывающие влиянии продольной и поперечной кривизны, внешнего градиента температуры и скорости, решены точно и приближенно. Рассматривается применение данной теории для случая переноса тепла от пучка эллиптических стержней и от сферы.